Math 401: Sec 0401: Homework 2

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Due: Sept. 19, 2014

Complete problems 1–5. Each of these problems is worth 20 points. In a question, each subproblem is worth the same amount of points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 1.3.23: Given the factorization

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -6 & 4 & -1 \\ 4 & -6 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & -4 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix},$$

explain, without computing, which elementary row operations are used to reduce A to upper triangular form. Be careful to state which order they should be applied. Then check the correctness of your answer by performing the elimination.

Problem 1.3.25: Let $t_1, t_2, \ldots$ be distinct real numbers. Find the LU factorization of the following *Vandermonde matrices*:

$$\text{(a)} \quad \begin{pmatrix} 1 & 1 \\ t_1 & t_2 \end{pmatrix} \qquad \text{(b)} \quad \begin{pmatrix} 1 & 1 & 1 \\ t_1 & t_2 & t_3 \\ t_1^2 & t_2^2 & t_3^2 \end{pmatrix}$$

Can you spot a pattern? Test your conjecture with the 4×4 Vandermonde matrix.

Problem 1.3.28: Prove that if A is a regular 2×2 matrix, then its LU factorization is unique. In other words, if $A = LU = \widehat{L}\widehat{U}$ where $\widehat{L}, L$ are lower triangular matrices with 1's on their diagonals (unit lower triangular matrices) and $\widehat{U}, U$ are upper triangular matrices, then $\widehat{L} = L$ and $\widehat{U} = U$. The general case will be analyzed in class.

Problem 1.4.9 : Write down the elementary 4×4 permutation matrices P_1 and P_2

- P_1 that permutes the second and fourth rows, and
- P_2 that permutes the first and fourth rows.
- Do P_1 and P_2 commute?
- Explain what the matrix products P_1P_2 and P_2P_1 do to a 4×4 matrix.

Problem 1.4.19.e: Find a permuted LU factorization of the matrix A, and use this factorization to solve the system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 4 & -1 & 2 \\ 7 & -1 & 2 & 3 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} -1 \\ -4 \\ 0 \\ 5 \end{pmatrix}$$