Math 401: Sec 0401: Homework 3

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Due: Sept. 29, 2014

Complete problems 1–5. Each of these problems is worth 20 points. In a question, each subproblem is worth the same amount of points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 1. Let $A \in \mathbb{R}^{3\times 3}$ be given by

$$A = \begin{pmatrix} 1 & a & 1 \\ a & 1 & 1 \\ b & ab+1 & 0 \end{pmatrix}, \qquad a, b \in \mathbb{R}.$$

a) For which values of a and b the matrix A is regular?

b) Set $a = -1$ and $b = 1$. Perform the permuted LU decomposition of A.

Problem 1.5.17. Prove that if $U \in \mathbb{R}^{n\times n}$ is a nonsingular upper triangular matrix, then the diagonal entries of U^{-1} are the reciprocals of the diagonal entries of U.

Problem 1.5.24.h. Find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & -2 & -5 \end{pmatrix},$$

by applying the *Gauss-Jordan method*.

Problem 1.6.13. Let $A, B \in \mathbb{R}^{m\times n}$.

(a) Suppose that $\mathbf{v}^T A\mathbf{w} = \mathbf{v}^T B\mathbf{w}$ for all vectors $\mathbf{v} \in \mathbb{R}^m$ and $\mathbf{w} \in \mathbb{R}^n$. Prove that $A = B$.

(b) Give an example of two matrices A and B such that $\mathbf{v}^T A\mathbf{v} = \mathbf{v}^T B\mathbf{v}$ for all vectors $\mathbf{v}$, but $A \neq B$.

Problem 5. Let $U = (u_{ij})_{i,j=1}^n$ be a nonsingular upper triangular matrix, and consider the linear system $U\mathbf{x} = \mathbf{b}$. This system can be solved using the following *Back Substitution* algorithm:

For $i = n, n-1, \dots, 1$

$$x_i = \frac{1}{u_{ii}} \left(b_i - \sum_{j=i+1}^{n} u_{ij} x_j \right);$$

end

(a) Perform an operation count for the algorithm above.

(b) Let $A \in \mathbb{R}^{n\times n}$ be a regular matrix. Perform an operation count to compute the solution of the linear system $A\mathbf{x} = \mathbf{b}$ using the LU decomposition of A.