Math 401: Sec 0401: Homework 4

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Due: Oct. 10, 2014

Complete problems 1–3. In a question, each subproblem is worth the same amount of points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 1.8.4. Let

$$A = \left(\begin{array}{ccc|c} a & 0 & b & 2 \\ a & 2 & a & b \\ b & 2 & a & a \end{array}\right)$$

be the augmented matrix for a linear system. For which values of a and b does the system have

i) a unique solution?

ii) no solution?

iii) infinitely many solutions?

Problem 2. Determine which of the following sets are vector spaces. If the set is not a vector space explain which property is not satisfied. On the other hand, if the set is a vector space you should provide a proof.

a) $V = \{A \in \mathbb{R}^{n\times n} : A \text{ is a diagonal matrix}\}$.

b) $V = \left\{A \in \mathbb{R}^{n\times n} : A = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}, \quad a, b \in \mathbb{R}\right\}$.

c) $V = \{(x, y, z) \in \mathbb{R}^3 : x \geq 0, y \geq 0, z \geq 0\}$.

d) $V = \{(x, y, z) \in \mathbb{R}^3 : yz = 0\}$.

Problem 2.2.13. Which of the following are subspaces? Justify your answer!

a) The set of all row vectors of the form $(a, 3a)$.

b) The set of all vectors of the form $(a, a + 1)$.

c) The set of all continuous functions in $\mathbb{R}$ for which $f(-1) = 0$.

d) The set of all periodic functions of period 1, i.e. $f(x + 1) = f(x)$.

e) The set of all non-negative functions, i.e. $f(x) \geq 0$.

f) The set of all even polynomials, i.e. $p(x) = p(-x)$.

g) The set of all polynomials $p(x)$ that have $(x - 1)$ as a factor.

h) The set of all quadratic forms $q(x, y) = ax^2 + bxy + cy^2$.