Math 401: Sec 0401: Homework 5

Enrique Otárola

Handed out: Nov. 3, 2014
Due: Nov. 10, 2014

Complete problems 1–5. In a question, each subproblem is worth the same amount of points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 3.1.2: Which of the following formulas for $\langle \boldsymbol{v}, \boldsymbol{w} \rangle$ define inner products on $\mathbb{R}^2$?

1. $2v_1w_1 + 3v_2w_2$.
2. $v_1w_2 + v_2w_1$.
3. $(v_1 + v_2)(w_1 + w_2)$.
4. $\sqrt{v_1^2 + v_2^2}\sqrt{v_1^2 + v_2^2}$.
5. $2v_1w_1 + (v_1 - v_2)(w_1 - w_2)$.
6. $4v_1w_1 - 2v_1w_2 - 2v_2w_1 + 4v_2w_2$.

Problem 3.1.11:

1. Prove the identity $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = \dfrac{1}{4}\left(\|\boldsymbol{u} + \boldsymbol{v}\|^2 - \|\boldsymbol{u} - \boldsymbol{v}\|^2\right)$, which allows us to reconstruct an inner product from its norm.
2. Use it to find the inner product on $\mathbb{R}^2$ corresponding to the norm $\|\boldsymbol{v}\| = \sqrt{v_1^2 - 3v_1v_2 + 5v_2^2}$.

Problem 3.1.26: Let $V = C^1(-1, 1)$ denote the vector space of continuously differentiable functions for $-1 \leq x \leq 1$.

1. Does the expression $\langle f, g \rangle = \int_{-1}^{1} f'(x)g'(x)dx$ define an inner product on V?
2. Answer the same question for the subspace $W = \{f \in V : f(0) = 0\}$ consisting of all continuously differentiable functions which vanish at 0.

Problem 3.2.19: Determine a basis for the subspace $W \subset \mathbb{R}^4$ consisting of all vectors which are orthogonal to the vector $(1, 2, -1, 3)^T$.

Problem 3.2.26:

1. Show that the polynomials $p_1(x) = 1, p_2(x) = x - \frac{1}{2}, p_3(x) = x^2 - x + \frac{1}{6}$ are mutually orthogonal with respect to the L^2-inner product on the interval $[0, 1]$.
2. Show that the functions $\sin(n\pi x), n = 1, 2, \ldots$ are mutually orthogonal with respect to the L^2-inner product on the interval $[0, 1]$.