Math 401: Sec 0401: Homework 6

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Complete problems 1–5. In a question, each subproblem is worth the same amount of points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 3.3.10: Which of the following formulas define norms on $\mathbb{R}^3$?

1. $\|\boldsymbol{v}\| = \sqrt{2v_1^2 + v_2^2 + 3v_3^2}$.
2. $\|\boldsymbol{v}\| = \sqrt{v_1^2 + 2v_1v_2 + v_2^2 + v_3^2}$.
3. $\|\boldsymbol{v}\| = \max\{|v_1|, |v_2|, |v_3|\}$.
4. $\|\boldsymbol{v}\| = |v_1 - v_2| + |v_2 - v_3| + |v_3 - v_2|$.
5. $\|\boldsymbol{v}\| = |v_1| + \max\{|v_2|, |v_3|\}$.

Problem 3.3.19: Let $\|\cdot\|_1$ and $\|\cdot\|_2$ be two different norms on a vector space V.

1. Prove that $\|\boldsymbol{v}\| = \max\{\|\boldsymbol{v}\|_1, \|\boldsymbol{v}\|_2\}$ defines a norm on V.
2. Does $\|\boldsymbol{v}\| = \min\{\|\boldsymbol{v}\|_1, \|\boldsymbol{v}\|_2\}$ define a norm?
3. Does the arithmetic mean
$$\|\boldsymbol{v}\| = \frac{1}{2}\left(\|\boldsymbol{v}\|_1 + \|\boldsymbol{v}\|_2\right)$$
define a norm on V?
4. Does the geometric mean
$$\|\boldsymbol{v}\| = \sqrt{\|\boldsymbol{v}\|_1\|\boldsymbol{v}\|_2}$$
define a norm on V?

Problem 3.4.25: Find the Gram matrix K for the functions $1, e^x, e^{2x}$ using the L^2 inner product on $[0, 1]$. Is K positive definite?

Problem 3.5.11:

- Prove that id K_1 and K_2 are positive definite $n \times n$ matrices, then $K = \begin{pmatrix} K_1 & \mathbf{0} \\ \mathbf{0} & K_2 \end{pmatrix}$ is a positive definite $2n \times 2n$ matrix.
- Is the converse true?

Problem 3.5.2–3.5.3:

1. Find an LDL^T factorization of the following symmetric matrix

$$A = \begin{pmatrix} 3 & -1 & 3 \\ -1 & 5 & 1 \\ 3 & 1 & 5 \end{pmatrix}.$$

   Determine whether A is positive definite or not.

2. For which values of c is the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & c & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

   positive definite?