LECTURE 1 | 09/03/14.

MATH 401. Linear Algebra and Applications.

i) ME = ENRIQUE.
ii) Syllabus.
iii) Schedule, book and topics.
iv) Grading Scheme.

§1. Linear Algebraic Systems.

§1.1. Solution of Linear Systems.

Consider the following linear system:

$$(1.1)\begin{cases} x + 2y + z = 2 \\ 2x + 6y + z = 7 \\ x + y + 4z = 3. \end{cases}$$

→ linear system of 3 eqns and 3 unknowns.

Why we are interested in solving linear systems of eqns?

i) pure mathemal interest!
ii) applications.

$$\underbrace{\boxed{\text{Model}}}\;\rightarrow\;\underbrace{\boxed{\text{PDE}}}_{\text{heat eqn.}}\;\rightarrow\;\underbrace{\boxed{\text{NUMERICS}}}_{\text{FEM}}\;\rightarrow\;\underbrace{\boxed{\text{LINEAR SYSTEM}}}_{\text{Tridiagonal linear system.}}$$

Solving linear systems of eqns is "a fundamental step" in applied mathematics.

→ Method: Gaussian elimination (invented in China 2000 years ago, attributed to Gauss)

→ Linear operation # 1: add a multiple of one equation to another equation.

Remark. Applications of # 1 lead to equivalent systems. This means, systems with the same solution.

Ex:
$$\left\{\begin{array}{l} x+2y+z=2 \\ 2x+6y+z=7 \\ x+y+4z=3 \end{array}\right. \quad \begin{array}{c} \xrightarrow{-2R_1+R_2} \\ \xrightarrow{-R_1+R_2} \end{array} \quad \begin{array}{l} x+2y+z=2 \\ 0\cdot x+2y-z=3 \\ 0\cdot x-y+3z=1 \end{array}$$

$$\xrightarrow{\frac{1}{2}R_2+R_3} \left\{\begin{array}{l} x+2y+z=2 \\ 2y-z=3 \\ \frac{5}{2}z=5/2 \end{array}\right. \qquad (1.2).$$

Consequence: We have obtained an equivalent system which is easier to solve!

$$(1.1) \Longleftrightarrow (1.2)$$

§ 1.2. Matrices and vectors.

Def (Matrix) A matrix is a rectangular array with m rows and n columns that looks like:

$$A = \begin{array}{cc} \begin{pmatrix} a_{11} & \cdots & \cdots & a_{1n} \\ a_{21} & \cdots & \cdots & a_{2n} \\ \vdots & & a_{ij} & \\ \vdots & & & \\ a_{m1} & \cdots & \cdots & a_{mn} \end{pmatrix} & \begin{array}{c} \\ \\ i \\ \\ \\ \end{array} \\ \qquad\qquad j & \end{array}$$

$a_{ij} \in \mathbb{R}$, $i = 1, \dots, m$ and $j = 1, \dots, n$

Notation $A \in \mathbb{R}^{m \times n}$: matrices of m rows and n columns with real numbers.

Examples:

$$A = \begin{pmatrix} 1 & 0 & 3 \\ -2 & 4 & 1 \end{pmatrix} \in \mathbb{R}^{2 \times 3}$$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \in \mathbb{R}^{2 \times 2}$$

$$A = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in \mathbb{R}^{2 \times 1}.$$

A vector is a particular case of a matrix.
To be precise:

We notice that System (1.2) is in triangular form: 

How do we solve triangular systems?

<u>Back Substitution</u>: From (1.2) we go backwards.

3rd eqn: $\boxed{z = 1}$

2nd eqn: $2y = 3 + z = 4$

$\boxed{y = 2}$

1st eqn: $x = 2 - 2y - z = 2 - 4 - 1$

$\boxed{x = -3}$

Exercise: Check $(x, y, z) = (-3, 2, 1)$ solves system (1.2) and (1.1)

Remark (existence and uniqueness)

The system (1.1) has a solution and is unique.

Bad news: this is not always true!

Special cases:

i) Square matrix : $m = n$

ii) Column vector : $n = 1$

Ex : $x = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$

iii) Row vector : $m = 1$

$x = \begin{pmatrix} x_1 & \cdots & x_n \end{pmatrix}$

End of lecture 1