

LECTURE 1 | 09/03/14.

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MATH 401. Linear Algebra and Applications.

- i) HE = ENRIQUE.
- ii) Syllabus.
- iii) Schedule, book and topics.
- iv) Grading Scheme.

§1. Linear Algebraic Systems.

§1.1. Solution of Linear Systems.

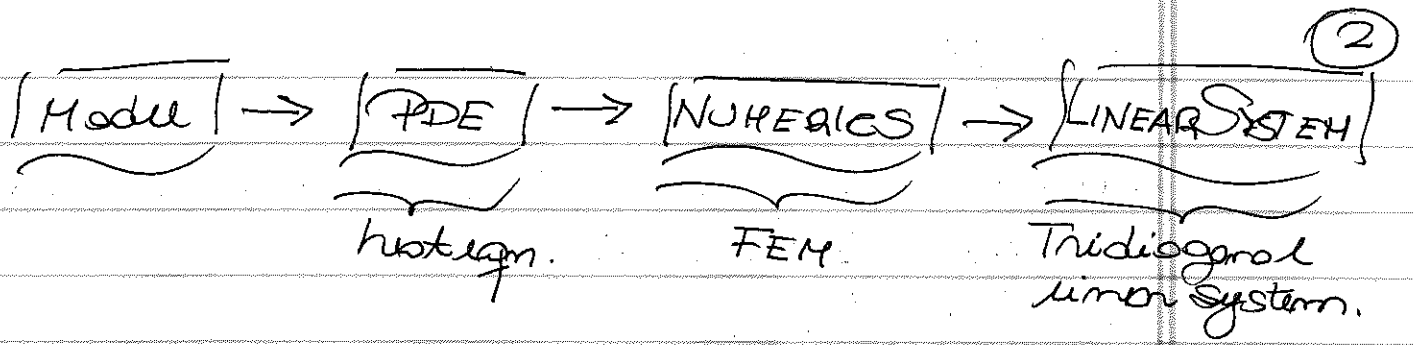
Consider the following linear system:

$$(A.1) \begin{cases} x + 2y + z = 2 \\ 2x + 6y + z = 7 \\ x + y + 4z = 3. \end{cases}$$

→ linear system of 3 eqns and 3 unknowns.

Why we are interested in solving linear systems of eqns?

- i) pure mathematical interest!
- ii) applications.



Solving linear systems of eqns is "a fundamental step" in applied mathematics.

→ Method: Gaussian elimination (invented in China 2000 years ago, attributed to Gauss)

→ Linear operation # 1: add a multiple of one equation to another equation.

Remark. Applications of # 1 lead to equivalent systems. This means, systems with the same solution.

$$\text{Exo. } \begin{cases} x + 2y + z = 2 \\ 2x + 6y + z = 7 \\ x + y + 4z = 3 \end{cases} \xrightarrow{-2R_1 + R_2, -R_1 + R_3} \begin{cases} x + 2y + z = 2 \\ 0x + 2y - z = 3 \\ 0x - y + 3z = 1 \end{cases}$$

$$\xrightarrow{\frac{1}{2}R_2 + R_3} \begin{cases} x + 2y + z = 2 \\ 2y - z = 3 \\ \frac{5}{2}z = \frac{5}{2} \end{cases} \quad (1.2)$$

Conclusion: We have obtained an equivalent system which is easier to solve!

$$(1.1) \Leftrightarrow (1.2)$$

§ 1.2. Matrices and vectors.

Def (Matrix) A matrix is a rectangular array with m rows and n columns that looks like:

$$A = \begin{pmatrix} a_{11} & \dots & a_{1m} \\ a_{21} & \dots & a_{2m} \\ \vdots & \dots & \vdots \\ a_{mm} & \dots & a_{mm} \end{pmatrix}$$

$$a_{ij} \in \mathbb{R}, \quad i = 1, \dots, m \text{ and } j = 1, \dots, m$$

Notation $A \in \mathbb{R}^{m \times m}$: matrices of m rows and m columns with real numbers.

Examples:


$$A = \begin{pmatrix} 1 & 0 & 3 \\ -2 & 4 & 1 \end{pmatrix} \in \mathbb{R}^{2 \times 3}$$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \in \mathbb{R}^{2 \times 2}$$

$$A = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in \mathbb{R}^{2 \times 1}$$

0-vector is a particular case of a matrix
To be precise:

(3)

We notice that system (1.2) is in triangular form: 

How do we solve triangular systems?
Back Substitution: From (1.2) we go backwards.

$$\text{3rd eqn: } |z = 1|$$

$$\text{2nd eqn: } \underbrace{2y = 3 + z = 4}_{y = 2}$$

$$\text{1st eqn: } \underbrace{x + 2 - 2y - z = 2 - 4 - 1}_{x = -3}$$

EXERCISE: Check $(x, y, z) = (-3, 2, 1)$ solves system (1.2) and (1.1)

Remark (existence and uniqueness)
The system (1.1) has a solution and is unique.

Bad news: this is not always true!

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Special Cases:

i) Square matrix: $m = n$

ii) Column vector: $m = 1$

$$\text{Ex: } x = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$$

iii) Row vector: $m = 1$

$$x = (x_1 \dots x_m)$$

end of lecture 1