

## Math 401 Section 0401: Quiz 3

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Oct. 19, 2014

Complete problems 1–2. Each of these problems is worth 5 points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

**Problem 1: (5 pts).** Given the rectangular matrix  $A$  and its row echelow form  $U$

$$A = \begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & 2 & -3 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 3 & -5 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

- (a) Determine a basis for the range (column space)  $\text{rng}(A)$  of  $A$  and the dimension of  $\text{rng}(A)$ .
- (b) Determine a basis for the kernel (null space)  $\text{ker}(A)$  of  $A$  and the dimension of  $\text{ker}(A)$ .

**Problem 2: (5 pts).**

- (a) Let  $V$  the space of integrable functions on  $[0, 1]$ . Show that the set of functions with integral zero is form a subspace of  $V$ .
- (b) Do the polynomials  $1 + 2x$  and  $x^2 - x + 2$  spand the space of polynomials  $\mathbb{P}^2$  of degree  $\leq 2$ ?
- (c) Do the the polynomials  $2x + 1$  and  $-x - 2$  form a basis of  $\mathbb{P}^1$ ?