Math 401 Section 0401: Quiz 4

Enrique Otárola

Nov. 12, 2014

Complete problems 1–2. Each of these problems is worth 5 points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 1: (5 pts). Let $\omega(x) > 0$ for $a \leq x \leq b$ be a weight function. Prove that

$$\|f\|_{1,\omega} := \int_a^b |f(x)|\omega(x)dx,$$

defines a norm on $C^0[a, b]$, called the *weighted* L^1 norm.

Problem 2: (5 pts). Use the L^2 inner product

$$\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)dx$$

to answer the following:

1. Find the *angle* between the function 1 and x. Are they orthogonal?
2. Find all quadratic polynomials $p(x) = a + bx + cx^2$ that are orthogonal to both functions 1 and x.