Math 401 Section 0401: Quiz 1

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Complete problems 1–2. Each of these problems is worth 5 points. Explain your steps carefully. If you use a *well known* theorem, make clear which theorem you are using and justify its use.

Problem 1: (5 pts). Find the LU factorization for the matrix A involved in the following linear system:

$$\begin{aligned} 2x - y &= 10 \\ 2x - 5z &= 1 \\ 6x - y - 8z &= 2. \end{aligned}$$

Write explicitely the factors L and U.

Solution: $A = \begin{pmatrix} 2 & -1 & 0 \\ 2 & 0 & -5 \\ 6 & -1 & -8 \end{pmatrix} \xrightarrow{-R_1+R_2} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -5 \\ 6 & -1 & -8 \end{pmatrix} \xrightarrow{-3R_1+R_3} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -5 \\ 0 & 2 & -8 \end{pmatrix} \xrightarrow{-2R_2+R_3} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 2 \end{pmatrix}$

$\Rightarrow U = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 2 \end{pmatrix}$ and $L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ ↑ look at the row operations!

Problem 1: (5 pts). Given the following LU factorization of A and the vector $\mathbf{b}$, find the solution $\mathbf{x}$ to $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 2 \end{pmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}.$$

Hint: Do not multiply L and U to get A.

Solution: $L\underset{\sim}{U}\underset{\sim}{x} = \underset{\sim}{b}$

First, $U\underset{\sim}{x} = \underset{\sim}{y}$ and $L\underset{\sim}{y} = \underset{\sim}{b}$

$L\underset{\sim}{y} = \underset{\sim}{b} \Leftrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} \Rightarrow \boxed{\begin{array}{l} y_1 = 1 \\ y_2 = 0 \\ y_3 = 2 \end{array}}$

Now, $U\underset{\sim}{x} = \underset{\sim}{y} \Leftrightarrow \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \Rightarrow \boxed{\begin{array}{l} x_3 = 1 \\ x_2 = 5 \\ x_1 = 3 \end{array}}$